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AGM Method Applied to Solve Nonlinear Electrical Circuits

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ABSTRACT

In this literature, two complicated nonlinear differential equations in the field of vibration have been analyzed and solved completely by algebraic method and we have named it Akbari-Ganji's Method (AGM) As regards the previous published papers, investigating this kind of equations is a very hard task to do and the obtained solution is not accurate and reliable. This issue will be emerged after comparing the achieved solutions by Numerical Method (Num.Rk 45) or the Exact Solution. Based on the comparisons which have been made between the gained solutions by AGM and Numerical Method, it is possible to indicate that AGM can be successfully applied for various differential equations particularly for difficult ones. Furthermore, it is necessary to mention that a summary of the excellence of this method in comparison with the other approaches can be considered as follows: Boundary conditions are needed in accordance with the order of differential equations in the solution procedure but when the number of boundary conditions is less than the order of the differential equation, this approach can create additional new boundary conditions in regard to the own differential equation and its derivatives. Therefore, it is logical to mention that AGM is operational for miscellaneous nonlinear differential equations in comparison with the other methods.

Keywords: Akbari-Ganji's Method (AGM), Oscillating System, RLC Nonlinear Electrical Circuit

Nomenclature

R	resistance
ω	Angular frequency
i	Current
V	Supply voltage
C	Capacitor
K_1	, K_2 Constant

Introduction

Along with the rapid progress of nonlinear sciences, an intensifying interest amongst scientists and researchers has been emerged in the field of analytical asymptotic techniques particularly for nonlinear problems. Although finding the solutions of linear equations by means of computer is very convenient, it is still very difficult and a time-consuming procedure to solve nonlinear problems either numerically or theoretically. Perhaps this is related to the fact that the various discredited methods or numerical simulations apply iteration techniques to find their numerical solutions of nonlinear problems and nearly all iterative methods are sensitive to initial solutions, so it is very difficult to obtain converged results in cases of strong nonlinearity [1,2]. In addition, the most important information such as the natural circular frequency of a nonlinear oscillation depends on the initial conditions (i.e. amplitude of oscillation) will be lost during the procedure of numerical simulation. Perturbation methods provide the most versatile tools available in nonlinear analysis of engineering problems and they are constantly being developed and applied to ever more complex problems. But like other nonlinear asymptotic techniques, perturbation methods have their own limitations take for example almost all perturbation methods are based on such an assumption that a small parameter must exist in an equation

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[3,4]. This so-called small parameter assumption greatly restricts applications of perturbation techniques, as is well known, an overwhelming majority of nonlinear problems, especially those having strong nonlinearity, have no small parameters at all and so on. Based on the above explanations, we should introduce some new developed methods for solving complicated nonlinear problems in different fields of study particularly in vibrations where traditional techniques have not been successful up to now. Furthermore, some techniques like perturbation methods are not practical for strongly nonlinear equations. As a result due to conquer these weak-points, in recent years, much attention has been devoted to the newly developed manners to gain an approximate solution of nonlinear equations such as : Energy Balance Method, Homotopy Analysis Method, He's Amplitude Frequency Formulation method (HAFF), Parameter-Expansion Method, EXP-function Method, Differential Transformation Method (DTM), Homotopy Perturbation Method, Adomian Decomposition Method and Variational Iteration Method by J.H.He [5-18]. But the afore-mentioned methods do not have this ability to gain the solution of the presented problem in high precision. Therefore, these complicated nonlinear equations such as the presented problems in this paper should be solved by utilizing other approaches like AGM[19-23].It is expected that other problems in the field of electronics described by nonlinear differential equations can be solved in a similar way, following the techniques employed in this work[24-26].Therefore, these complicated nonlinear equations such as the presented problems in this paper should be solved by utilizing other approaches like AGM [27-34].And in recent years, other new methods have been invented by Mr. Mohammad Reza Akbari, whose analytical solution accuracy of complex problems is very high, and always convergent, easy to understand and use for users, and very flexible,like methods AKLM (Akbari Kalantari Leila Method),ASM (Akbari Sara's Method),AYM (Akbari Yasna's Method), IAM (Integral Akbari Method) [35-41].

The Analytical Method (Akbari-Ganji's Method (AGM))

In general, vibrational equations and their initial conditions are defined for different systems as follows:

$$f\left(\ddot{u},\dot{u},u,F_0\sin(\omega_0 t)\right)=0 \tag{1}$$

Parameter (ω_0) angular frequency of the harmonic force exerted on the system and (F_0) the maximum amplitude its. And initial conditions as follows

$$\{u(t)=u_0 \quad , \quad \dot{u}(t)=0 \quad , \quad at \quad t=0\} \tag{2}$$

Choosing the Answer of the Governing Equation for Solving Differential Equations by AGM

In AGM, a total answer with constant coefficients is required in order to solve differential equations in various fields of study such as vibrations, structures, fluids and heat transfer. In vibrational systems with respect to the kind of vibration, it is necessary to choose the mentioned answer in AGM. To clarify here, we divide vibrational systems into two general forms:

Vibrational Systems Without Any External Force

Differential equations governing on this kind of vibrational systems are introduced in the following form:

$$f(\ddot{u},\dot{u},u)=0 \tag{3}$$

Now, the answer of this kind of vibrational system is chosen as:

$$u(t)=e^{-bt}\{A\cos(\omega t)+B\sin(\omega t)\} \tag{4}$$

According to trigonometric relationships, Eq. (4) is rewritten as follows:

$$u(t)=e^{-bt}\{a\cos(\omega t+\varphi)\} \tag{5}$$

It is notable that in the Eq. (5),

$$a=\sqrt{A^2+B^2}\ ,\ \varphi=\arctan(\frac{B}{A})$$

Sometimes for increasing the precision of the considered answer of Eq. (3), we are able to add another term in the form of cosine by inspiration of Fourier cosine series expansion as follows:

$$u(t)=e^{-bt}\{a\cos(\omega t+\varphi_1)+d\cos(2\omega t+\varphi_2)\} \tag{6}$$

In the above equation, we are able to omit the term (e^{-bt}) to facilitate the computational operations in AGM if the system is considered without any damping components.

Generally speaking in AGM, Eq. (5) or Eq. (6) is assumed as the answer of the vibrational differential equation (3) that its constant coefficients which are *a, b, c, ω* (angular frequency) and *φ* (initial vibrational phase) can easily be obtained by applying the given initial conditions in Eq. (2). And also, the above procedure will completely be explained through the presented example in the foregoing part of the paper. It is noteworthy that if there is no damping component in the vibrational system, the constant coefficient b in Eq. (5) and Eq. (6) will automatically be computed zero in AGM solution procedure. On the contrary, the parameter b in Eq. (5) and Eq. (6) for the other kind of vibrational system with damping component is obtained as a nonzero parameter in AGM.

Vibrational Systems with External Force

In this step, it is assumed that the external forces exerting on the vibrational systems are defined as:

$$F(t)=F_0\sin(\omega_0 t) \tag{7}$$

As a result, the differential equation governing on the vibrational system is expressed like Eq. (1) as follows:

$$f\left(\ddot{u},\dot{u},u,F_0\sin(\omega_0 t)\right)=0 \tag{8}$$

The answer of the above equation is introduced as the sum of the particular solution (u_p) and the harmonic solution (u_h) as follows:

$$\begin{aligned} u_h(t)&=e^{-bt}\{A\cos(\omega t)+B\sin(\omega t)\}\\ u_p(t)&=M\cos(\omega_0 t)+N\sin(\omega_0 t)\end{aligned} \tag{9}$$

The result answer differential equation Eq. (8) as follows:

$$u(t)=u_p+u_h \tag{10}$$

By utilizing trigonometric relationships $\{a=\sqrt{A^2+B^2}\ ,\ d=\sqrt{M^2+N^2}\ \}$ and $\{\ \varphi=\arctan(\frac{B}{A})\ ,\phi=\arctan(\frac{N}{M})\}$ also by substituting the yielded equations into Eq. (10), the desired answer will be obtained in the form of:

$$u(t)=e^{-bt}\{a\cos(\omega t+\varphi)\}+d\cos(\omega_0 t+\phi) \quad (11)$$

In order to increase the precision of the achieved equation, we are able to add another term in the form of cosine by inspiration of Fourier cosine series expansion as follows:

$$u(t)=e^{-bt}\{a.\cos(\omega t+\varphi_1)+c.\cos(2\omega t+\varphi_2)\}+d.\cos(\omega_0 t+\phi) \quad (12)$$

And finally in accordance with the Eq. (12), the exact solution of the all-vibrational differential equations can be obtained in the following equation:

$$u(t)=e^{-bt}\{\sum_{k=1}^{\infty}a_k\cos(k\omega t+\varphi_k)\}+d\cos(\omega_0 t+\phi) \quad (13)$$

The constant coefficients of Eq. (13) which are $\{a_1,a_2,\dots,\varphi_1,\varphi_2,\dots,b,\omega,d,\varphi\}$ will easily be computed in AGM by applying the initial conditions of Eq. (2). To deeply understand the above procedure, reading the following lines is recommended. Since the constant coefficient (*b*) in vibrational systems without damping components is always obtained zero (b=0) which in the case, to decrease computational operations of Eq. (12) and (or Eq. (13)) in the following form:

$$\begin{Bmatrix} u(t)=\{a.\cos(\omega t+\varphi_1)+c.\cos(2\omega t+\varphi_2)\}+d.\cos(\omega_0 t+\phi) \\ or \qquad u(t)=\{\sum_{k=1}^{\infty}a_k\cos(k\omega t+\varphi_k)\}+d\cos(\omega_0 t+\phi) \end{Bmatrix} \quad (14)$$

Based on the above explanations, by applying initial conditions on a system without damping component, the value of parameter (*b*) is always zero for Eq.s (11-13) .Therefore without damping component, the role of parameter (*b*) in the both of Eq.(11-13) which each of them can be considered as the answer of the vibrational problems is individually considered as a catalyst for increasing the precision of the assumed answer. However, according to Eq.s(11-13) after applying initial conditions on the vibrational system in both states (with external force and without external force) by AGM, the value of parameter (*b*) is computed zero because the mentioned system has a free vibration without any damping component. Again, we mention that in order to decrease computational operations for systems without damping components and since we know that (*b*) in the term (e^{-bt}) is zero so (e^{-bt}) can be omitted from Eq.s(11-13). Consequently, Eq. (11) which has been considered as the answer of the systems without any damping component can be rewritten as follows:

$$u(t)=a.\cos(\omega t+\varphi)+d.\cos(\omega_0 t+\phi) \quad (15)$$

Application of Initial Conditions to Compute Constant Coefficients and Angular Frequency by AGM

In AGM, the application of initial conditions of Eq. (2) is done in the two following forms:

Applying the initial conditions on the answer of differential equation

In regard to the kind of vibrational system (with external force and without external force) which was completely discussed in the previous part of this case study, a function is chosen as the answer of the differential equation from Eq.(5) or Eq.(6) for the systems without external forces and from Eq.s(11-15) for the defined systems with external forces and then the initial conditions are applied on the selected function as follows:

$$u(t) = u(IC) \quad (16)$$

It is notable that IC is the abbreviation of introduced initial conditions of Eq. (2).

Appling the initial conditions on the main differential equation and its derivatives

After choosing a function as the answer of differential equation according to the kind of vibrational system, this is the best time to substitute the mentioned answer into the main differential equation instead of its dependent variable (*u*).Assume the general equation of the vibration such as Eq.(1) with time-independent parameter (*t*) and dependent function (*u*) as:

$$f\left(\ddot{u},\dot{u},u,F_0\sin(\omega_0 t)\right)=0 \quad (17)$$

Therefore, on the basis of the kind of vibrational system, a function as the answer of the differential equation such as Eq. (5) or Eq. (6) and Eq.s (11-15) are considered as follows:

$$u = g(t) \quad (18)$$

In this step, the afore-mentioned equation is substituted into Eq. (17) instead of (*u*) in the following form:

$$f(t)=f(g''(t),\ g'(t),\ g(t),\ F_0\sin(\omega_0 t)) \quad (19)$$

Eventually, the application of initial conditions on Eq. (19) and its derivatives is expressed as:

$$\begin{aligned} &f(IC)=f(g''(IC),\ g'(IC),\ g(IC),\ \dots)\ , \\ &f'(IC)=f'(g''(IC),\ g'(IC),\ g(IC),\ \dots)\ , \\ &f''(IC)=f''(g''(IC),\ g'(IC),\ g(IC),\ \dots)\ ,\ \dots.. \end{aligned} \quad (20)$$

To end up, it is better to say that in AGM after applying the initial conditions on answer function Eq. (16), and also the function differential equation and on its derivatives from Eq.s (20) according to the order of differential equation and utilizing the two given initial conditions of Eq. (2), a set of algebraic equations which is consisted of n equations with n unknowns is created. Therefore, the constant coefficients *a, b, c, d,* angular frequency *ω* and initial phase φ and ϕ at Eq.s(11,12) are easily achieved which this procedure will thoroughly be explained in the form of an example in the foregoing part of this paper. It is noteworthy that in Eq. (19), we are able to use the derivatives of *f(t)* with higher orders until the number of yielded equations is equal to the number of the mentioned constant coefficients of the assumed answer.

Application, the First Order Nonlinear Differential Equations

The first order nonlinear differential equations can be considered in the following general form:

$$f(t)=f(u'(t),g(u(t)))=0 \quad (21)$$

And the relevant initial condition is expressed as:

$$u(0) = u_0 \tag{22}$$

In this step, a compound series which is consisted of exponential and trigonometric terms has been selected as a suitable answer for this kind of nonlinear differential equations as follows

$$u(t)=\sum_{k=0}^{n}\left\{a_k\, e^{-b_k t}+d_k \cos(\Omega t+\varphi_k)\right\}=\left\{(a_0\, e^{-b_0 t}+a_1\, e^{-b_1 t}+...)+(d_0 \cos(\Omega t+\varphi_0)+d_1 \cos(\Omega\, t+\varphi_1)+...)\right\} \tag{23}$$

Moreover, for analyzing electrical circuits in electrical engineering we can utilize the above relation. It is citable that Eq. (23) is applicable for solving all of the first order nonlinear differential equations. To understand more, it is better to indicate that to facilitate the computational operations in AGM if the physical system is not vibrational such as differential equations governing on the heat transfer systems (Radiation), Chemical reactions or each non-vibrational differential equations it is logical to omit the trigonometric term of Eq. (23). Therefore, Eq. (23) can be rewritten as follows:

$$u(t)=\sum_{k=0}^{n}\left\{a_k\, e^{-b_k t}\right\}=a_0 e^{-b_0 t}+a_1 e^{-b_1 t}+....+a_n e^{-b_n t} \tag{24}$$

In the afore-mentioned equations, Eq. (23) can be a suitable choice for the answer function. Based on the given explanations, the constant coefficients of Eqs. (23,24), {(a_0, b_0, d_0) and (φ_0 *to* φ_n)} can be obtained by applying the given initial condition and is the angular frequency of the system.

In order to show how to obtain approximate analytical solutions in some nonlinear circuits, we consider the case of dependence of the resistance R with the current i(t) [42]. We assume, for simplicity, the following second order expression.

$$R = R_0 + k_2 i(t) + k_2 i^2(t) \tag{25}$$

where R_0 and k_1 and k_2 are constant parameters

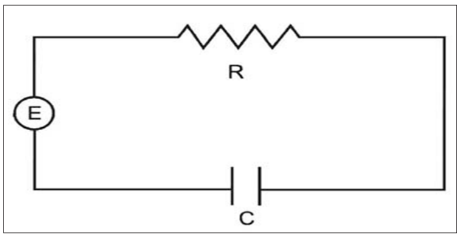

Figure 1: Basic RC circuit.

A basic RC alternate current circuit may consist of a constant resistor R0 and a capacitor C, connected in series with a source $V = V_0 \sin(\omega_0 t)$, as shown in Figure 1. Equation for electric current, is obtained from Kirchoff's second Law:

$$R.\mathrm{i}+\frac{1}{c}\int i\,dt=V_0 \sin(\omega_0\, t) \tag{26}$$

differentiating (26), we obtain

$$R.\frac{di}{dt}+\frac{i}{c}=V_0\,\omega_0\,\cos(\omega_0\, t) \tag{27}$$

Nonlinear effects are given in Eq (25), into (27) and differentiating to obtain follow:

$$(R_0+\mathrm{k}_1\mathrm{i}+\mathrm{k}_2\mathrm{i}^2)\frac{di}{dt}+\frac{i}{c}=V_0\,\omega\,\cos(\omega_0\, t) \tag{28}$$

Substituting values: ω_0, $R_2 = 5$, $V_0 = 1$, $c = 3$, $k_1=\frac{1}{5}$ and $k_2=\frac{1}{3}$ the equation results in a generalization of (28) is the following nonlinear differential equation in the form of:

$$\mathrm{f(t)}:\ (5+\frac{i(t)}{5}+\frac{i^2(t)}{3})\frac{d\,i(t)}{dt}+\frac{i(t)}{3}=2\cos(2\,t) \tag{29}$$

Then, the initial conditions are expressed as:

$$\mathrm{i}(0) = 0.1 \tag{31}$$

Solving the Nonlinear Differential Equation by AGM

On the basis of the given explanations in the previous section (the analytical method), the answer of Eq. (29) is considered by AGM as a polynomials of Fourier series with constant coefficients as follows:

$$i(t)=a\,e^{-b\,t}+d\cos(\omega_0\,t+\varphi),\ \omega_0=2 \tag{32}$$

It is notable that in AGM, the constant coefficients of Eq.(31) which are *a,b,d* and *φ* can easily be computed by applying initial or boundary conditions.

Applying Initial or Boundary Conditions in AGM

Based on the given explanations in the previous section of this paper, the constant coefficients *a,b,d* and *φ* from Eq. (31) are just achieved with respect to the given initial conditions and these initial conditions are applied in two manners in AGM.

With regard to Eq. (30), the initial conditions are applied on Eq. (31) as follows:

$$\mathrm{i}(0) = 0.1 \text{ so } a+b\cos\varphi = 0.1 \tag{32}$$

In accordance with Eq. (20), the application of initial conditions on the main differential equation which in this case is Eq. (29) and also on its derivative is done after substituting Eq. (31) which has been considered as the answer of the main differential equation into Eq. (29) as:

$$f(\mathrm{i}(0)):\quad -[5+\frac{1}{5}(a+d\cos\varphi)^2+\frac{1}{3}a+\frac{1}{3}d\cos\varphi](ab+2d\sin\varphi)+\frac{1}{3}(a+d\cos\varphi)=2 \tag{33}$$

Then for the first derivative of the achieved equation, we will have:

$$f'(\mathrm{i}(0)):\quad [\frac{2}{3}(a+d\cos\varphi)(ab-2d\sin\varphi)+\frac{1}{3}ab+\frac{2}{3}d\sin\varphi](ab+2d\sin\varphi)+ [5+\frac{1}{5}(a+d\cos\varphi)^2+\frac{1}{3}(a+d\cos\varphi)](ab^2-4d\cos\varphi)-\frac{1}{3}(ab-2d\sin\varphi)=0 \tag{34}$$

Then for the second derivative of the achieved equation, we will have:

$$f''(\mathrm{i}(0)):\quad \{\frac{2}{5}(ab+2d\sin\varphi)^2+\frac{2}{5}(a+d\cos\varphi)(\mathrm{a\,b}^2-4d\cos\varphi)+\frac{1}{3}ab^2-\frac{4}{3}d\cos\varphi\}(-ab-2d\sin\varphi) -2\,[\frac{2}{5}(a+d\cos\varphi)(ab+2d\sin\varphi+\frac{1}{3}\mathrm{a\,b}+\frac{2}{3}\mathrm{d}\sin\varphi)](ab^2-4d\cos\varphi)+\{5+\frac{1}{5}(a+d\cos\varphi)^2 \frac{1}{3}(a+d\cos\varphi)\}(-ab^3+8d\sin\varphi)+\frac{1}{3}(ab^2-4d\cos\varphi)=-8 \tag{35}$$

By solving a set of algebraic equations which is consisted of four equations with four unknowns from Eqs. (32-35), the constant coefficients of Eq. (31) can easily be yielded at follow:

$$a = 0.0906,\ b = 0.070344,\ d = 0.1987,\ \varphi = -1.5234 \qquad (36)$$

After substituting the obtained values from Eqs. (36) into Eq. (31), the answer of the presented problem is achieved in the following form:

$$\mathrm{i}(t) = 0.0906\, e^{-0.070344\, t} + 0.1987 \cos(2t - 1.5234) \qquad (37)$$

According to the afore-mentioned equation, it is possible to draw the obtained solution by AGM as follows:

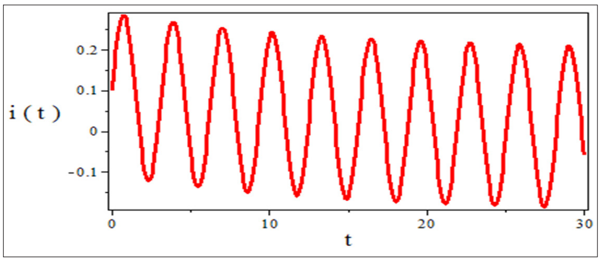

Figure 2: The chart of the obtained solution by AGM.

The introduced domain $t \in \{0,30\}$ which is defined in terms of second (sec), the Numerical Solution of the mentioned problem is presented in the table below:

Table 1: The results of Numerical Solution the based in the specified domain.

t (sec)	0.0	6	12	28	24	30
u (t) Num Rk 45	0.1	-0.038507	-0.0136094	-0.1711148	-0.138830	-0.0540188

Comparing the obtained solutions by AGM and Numerical Method

The following charts are compared on the basis of the yielded solution by AGM and the results of table 1 by Numerical Method:

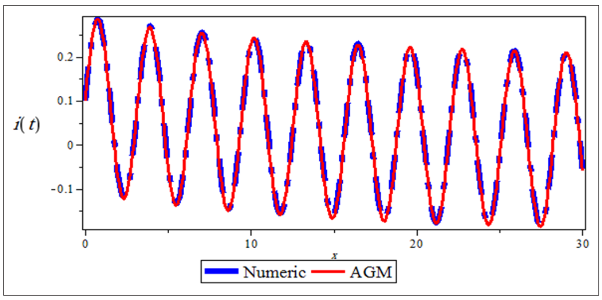

Figure 3: Comparing the obtained solutions by AGM and Numerical Method

In regard to the above charts, it is clear that AGM is a very applicable and reliable method for solving highly nonlinear vibrational differential equations with high precision.

Difference of the Obtained Solutions by AGM and Numerical Method

The following charts are difference on the basis of the yielded solution by AGM and the results of table 1 by Numerical Method:

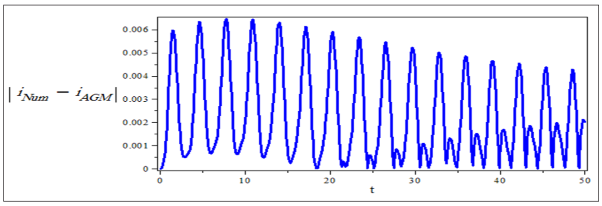

Figure 4: Difference the obtained solutions by AGM and Numerical Method.

Flow chart of the capacity of the capacitor and the initial intensity of the electric current

For various amounts of capacitor, take for example C =1, 3, 5, 7 and finally c =9, the resulted intensity of electric current are illustrated by AGM as:

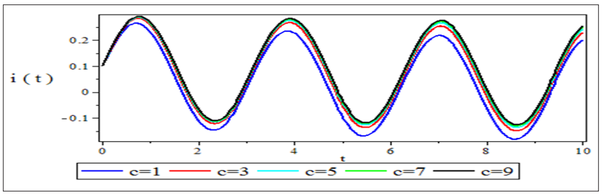

Figure 5: Comparing the obtained electric current by AGM on the basis of increasing the values of capacitor (C).

For various amounts of the initial intensity of the electric current, take for example i 0 =0.1, 0.3, 0.5, 0.7 and finally i 0 =0.9, the resulted intensity of electric current is illustrated by AGM as:

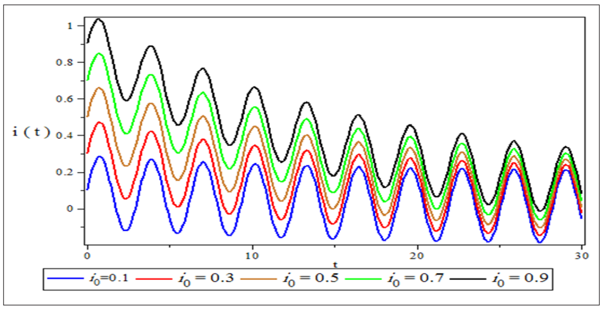

Figure 6: Comparing the obtained electric current by AGM on the basis of increasing the values of the initial intensity of the electric current (i_0).

Example 2

The well-known electrical circuit with a nonlinear element, which is represented as a nonlinear inductor, an alternating source of voltage, a pure resistive element and an electrical capacitor, is shown in Figure 1. Applying the node law of the circuit theory, such a circuit can be modeled as:

$$f(t) = \frac{d^2\Phi}{dt^2} + \frac{1}{RC}\left(\frac{d\Phi}{dt}\right) + \frac{\alpha_1}{C}\Phi + \frac{\alpha_3}{C}\Phi^3 = 0 \qquad (38)$$

where Φ is the magnetic ux through the nonlinear inductor, E_0 is the alternating source voltage, R and C are the constants of the capacitor, α_1 and α_2 are some operation constants. The nonlinear term appears because of the nonlinear inductor, which is an inductor with a ferromagnetic core, and is modeled, if an abstraction of the hysteresis phenomenon is made, by an *i-Φ* nonlinear characteristic. Here i is the electrical current. Such a characteristic is approximated by a constitutive relation of the form [43-45]:

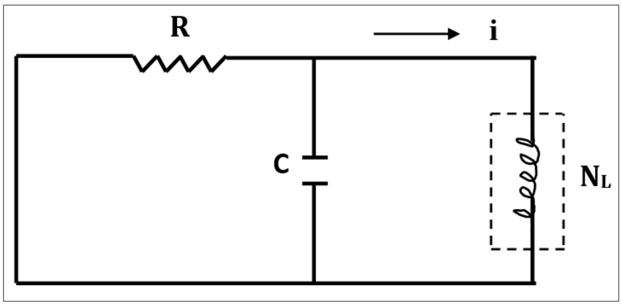

Figure 7: RLC circuit.

$$i=\alpha_1\Phi+\alpha_3\Phi^3 \tag{39}$$

Solving the differential equation by AGM

In order to solve Eq. (38), a finite series with constant coefficients has been considered as the answer of Eq. (38) as follows:

$$\Phi(t)=e^{-at}\{b\cos(\omega t+\varphi)\} \tag{40}$$

Constant coefficients *a, b, ω, φ* in the afore-mentioned equation will be computed by applying initial conditions.

Applying Initial Conditions in AGM

Exactly like the procedure which has been explained in example 1, the initial conditions are applied in two ways in AGM. Therefore, we have:

$$\Phi(0)=A \quad \text{so} \quad b\cos\varphi=A \tag{41}$$

Afterwards,

$$\dot{\Phi}(0)=0 \text{ therefore } b(a\cos\varphi+\omega\sin\varphi)=0 \tag{42}$$

Then, applying the boundary conditions on the main differential equation which is Eq. (38) and is named and on its derivatives is done after substituting Eq. (40) into differential equation (38) in the following form:

-applying initial conditions on the yielded equation:

$$f(\Phi(t=0)):$$
$$\text{so} \quad \frac{a}{R}(b^2RC-\omega^2RC-b+\alpha_1R)\cos\varphi+\frac{a\omega}{R}(2bRC-1)\sin\varphi+\alpha_2a^3\cos^3\varphi=0 \tag{43}$$

-applying initial conditions on the first derivative of the yielded equation:

$$f'(\Phi(t=0)):$$
$$\frac{a}{R}(3b\omega^2RC+b^2-b^3RC-\omega^2-\alpha_1bR)\cos\varphi - \frac{a\omega}{R}(3b^2RC-\omega^2RC-2b+\alpha_1R)\sin\varphi-3a^2\alpha_2(b\cos\varphi+\omega\sin\varphi)\cos^2\varphi=0 \tag{44}$$

In this step, by solving a set of algebraic equations which is consisted of four equations with four unknowns from Eqs. (41) to Eq. (44), the constant coefficients of Eq. (40) which are *a, b, φ and ω* will be obtained very easily as follows: To simplify, the following new variables are introduced as:

$$\psi=4C(\alpha_1+\alpha_2A^2) \tag{45}$$

$$a=\frac{1}{2RC},\quad b=AR\sqrt{\frac{\psi}{\psi-1}},\quad \varphi=tg^{-1}\left(\frac{1}{R\sqrt{\psi}}\right) \tag{46}$$

And with respect to the above procedure, ω (Electrical circuit angular frequency), can be computed in the forms of:

$$\omega=\frac{1}{RC}\sqrt{C(\alpha_1+\alpha_2A^2)} \tag{47}$$

After substituting the obtained values from Eqs. (46) and Eq. (47) into Eq. (40), the solution of the mentioned problem will be obtained as follows:

$$\Phi(t)=AR\sqrt{\frac{\psi}{\psi-1}}\,e^{-\frac{1}{2RC}t}\cos\left\{\frac{1}{2RC}\sqrt{\psi}\,t+tg^{-1}\left(\frac{1}{R\sqrt{\psi}}\right)\right\} \tag{48}$$

By selecting the physical values below:

$$A=0.1,\quad R=8,\quad C=4,\quad \alpha_1=0.2,\quad \alpha_2=0.1 \tag{49}$$

So, the answer and the related angular frequency of the presented problem, Eq. (38), are gained in the following form:

$$\omega=0.22362\left(\frac{Rad}{\sec}\right) \tag{50}$$

$$\Phi(t)=0.10024e^{-0.01562t}\cos(0.22362t+3.07183) \tag{51}$$

Consequently, the charts of the obtained solution and its derivative are depicted as follows:

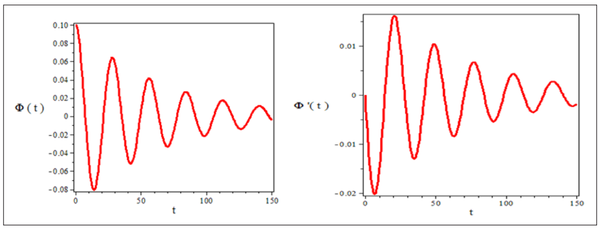

Figure 8: The chart of the obtained solution by AGM.

Figure 9: The chart of the first derivative for the obtain solution by AGM

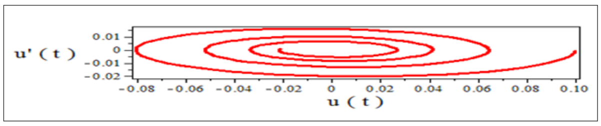

Figure 10: The resulted phase plane for Example2 by AGM

Numerical Solution of the Differential Equation

The achieved results of Numerical Solution are presented in the specified domain in follows:

Table 2: The results of Numerical Solution based on the given physical values in the specified domain

t (sec)	0.0	10	20	30	40	50
$\Phi(t)$ Num.Rk 45	0.1	0.0590134	0.02849704	0.009614978	-0.000112	-0.0038915
$\Phi'(t)$Num.Rk 45	0.0	-0.0056908	-0.0064779	-0.00519763	-0.003430	-0.0019031

Comparing the Achieved Solutions by AGM and Numerical Method

The following charts are depicted according to table 2 from Numerical Method and the obtained solution by AGM in order to compare the achieved solutions together in the following forms:

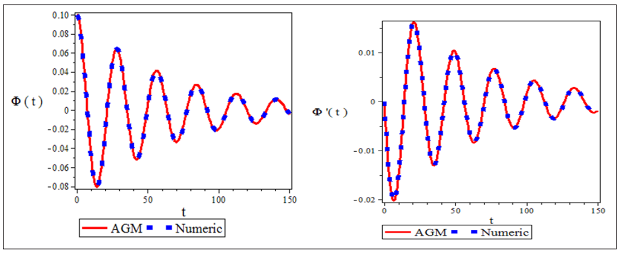

Figure 11: A comparison between the achieved solutions Figure 12: Comparing the first derivative of the obtained by AGM and Numerical method. solutions by AGM and Numerical method.

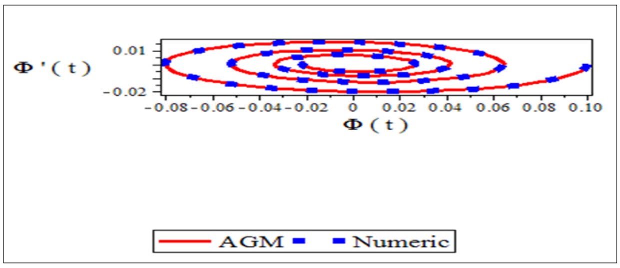

Figure 13: Comparing the related phase planes of the achieved solutions by AGM and Numerical Method.

Difference of the obtained solutions by AGM and Numerical Method

The following charts are difference on the basis of the yielded solution by AGM and the results of table 1 by Numerical Method:

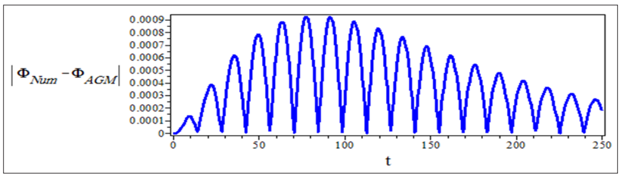

Figure 14: Difference the obtained solutions by AGM and Numerical Method

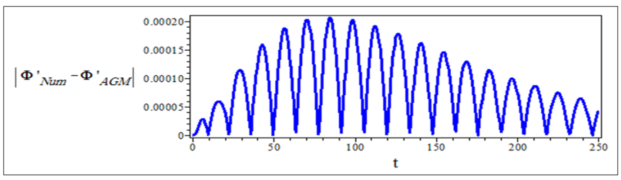

Figure 15: Difference of the first derivative the obtained solutions by AGM and Numerical Method

Comparing the charts of Phase Planes for different values of amplitude of vibration by AGM

For various amounts of capacitor, take for example R =2, 4, 6 and finally R =8, the resulted intensity of electric current are illustrated by AGM as:

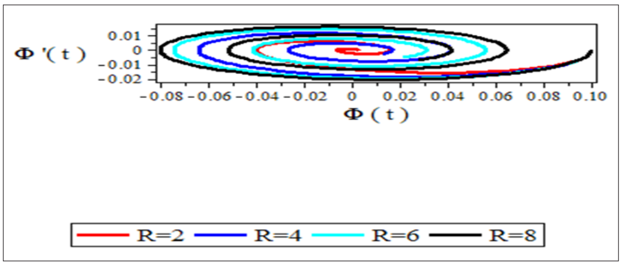

Figure 16: Comparing the obtained phase planes by AGM on the basis of increasing the values of resistance (R).

For various amounts of capacitor, take for example C =1, 3, 5 and finally c =7, the resulted intensity of electric current are illustrated by AGM as:

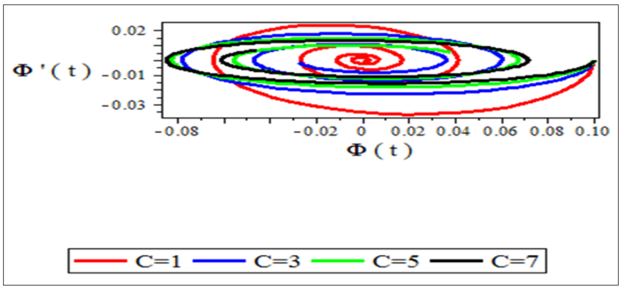

Figure 17: Comparing the obtained phase planes by AGM on the basis of increasing the values of capacitor (C).

Conclusions

In this paper, two complicated nonlinear vibrational differential equations (electric current) have been introduced and analyzed completely by Akbari-Ganji's Method (AGM)and also the obtained results have been compared with Numerical Method. Then, the vibrational velocity and vibrational acceleration have successfully been achieved. Afterwards, the related equations of locus for vibrational velocity and acceleration have been gained and depicted completely. Eventually, the equation of damping ratio in terms of initial amplitude of vibration and angular frequency has been obtained perfectly. The above process has been done in order to show the ability of AGM for solving a broad range of differential equations in different fields of study particularly in vibrations. Consequently, it is concluded that AGM is a reliable and precise approach for solving miscellaneous differential equations. Moreover, a summary of the AGM excellence and benefits is explained as: By solving a set of algebraic equations with constant coefficients, we are able to obtain the solution of nonlinear differential equation along with the related angular frequency simultaneously very easily which applying this procedure is possible even for students with intermediate mathematical knowledge. On the other hand, it is better to say that AGM is able to solve linear and nonlinear differential equations directly in most of the situations that means the final solution can be obtained without any dimensionless procedure. Therefore, AGM can be considered as a significant progress in nonlinear sciences.

History of AGM, ASM, AYM, AKLM, MR. AM and IAM,WoLF-a, SYM methods:

AGM (Akbari-Ganji Methods), ASM (Akbari-Sara's Method), AYM (Akbari-Yasna's Method) AKLM (Akbari Kalantari Leila Method), MR.AM (MohammadReza Akbari Method)and IAM (Integral Akbari Methods),WoLF,a method (Women Life Freedom,akbari), have been invented mainly by Mohammadreza Akbari (M.R.Akbari) in order to provide a good service for researchers who are a pioneer in the field of nonlinear differential equations.

- **AGM** method Akbari Ganji method has been invented mainly by Mohammadreza Akbari in 2014. Noting that Prof. Davood Domairy Ganji co-operated in this project.
- **ASM** method (Akbari Sara's Method) has been created by Mohammadreza Akbari on 22 of August, in 2019.
- **AYM** method (Akbari Yasna's Method) has been created by Mohammadreza Akbari on 12 of April, in 2020.
- **AKLM** method (Akbari Kalantari Leila Method) has been created by Mohammadreza Akbari on 22 of August, in 2020.
- **MR.AM** method (Mohammad Reza Akbari Method) has been created by Mohammadreza Akbari on 10 of November, in 2020.
- **IAM** method (Integral Akbari Method) has been created by Mohammadreza Akbari on 5 of February, in 2021.
- **Wolf- a** method (Women Life Freedom, akbari) has been created by Mohammadreza Akbari on 5 of February, in 2022.
- **SYM** method (Women Life Freedom, akbari) has been created by Mohammadreza Akbari on 5 of February, in 2023.

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