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Review Article

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Interdisciplinary Uses and Kernel-Based Enhancements of Line Transect Sampling

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Abstract
Line transect sampling is a widely used statistical framework for estimating population densities and abundances, particularly in wildlife and ecological studies. Traditionally relying on parametric detection functions, the methodology has evolved through nonparametric, kernel-based, histogram, and semi-parametric approaches, enhancing flexibility and robustness, especially when classical assumptions such as the shoulder condition are violated. This review synthesizes theoretical foundations, methodological developments, and interdisciplinary applications of line transect sampling, with emphasis on kernel-based innovations pioneered by Eidous and collaborators. Applications extend beyond ecology to reliability analysis, linear degradation models, environmental monitoring, epidemiology, industrial inspection, astronomy, and statistical approximation. Recent advances, including adaptive bandwidth selection, double-kernel methods, bias correction, and two-parameter detection functions, highlight ongoing developments and future research directions, demonstrating that line transect sampling constitutes a general framework for density estimation across the sciences.

Keywords: Line Transect Sampling, kernel Methods, Semi-Parametric Models, Histogram Estimator, Wildlife Density Estimation, Reliability Analysis, Statistical Approximation, Spatial Statistics, Interdisciplinary Applications

Introduction

One of the main goals in wildlife population studies is to estimate population abundance (density) D. Estimating *D* is a critical task in ecology, statistics, and reliability engineering. Line transect sampling, first introduced by and further developed by [1-4]. The approach involves traversing a line and recording perpendicular distances from the transect to detected objects, providing a statistically rigorous framework for density estimation while accounting for imperfect detection.

Assume that $g(x)$ is the detection function of x, which represents the probability of detecting an object given that it is perpendicular distance is x. The function $g(x)$ relies on primary two assumptions. Firstly, $g(x)$ should be monotonically decreasing in x, which indicates that the probability of detecting an object is decreases as its perpendicular distance increases. Secondly, $g(x)$ equals one at distance $x=0$ *(i.e,* $g(0)=1$*)*, which means that the probability of detecting an object on the line is certain and there is no missing objects on the line itself. Let X is a random variable represents the perpendicular distance of an object with detection functiong (x). Burnham and Anderson (1976) gave the relationship between $g(x)$ and the probability density function *(i.e* $f(x)$*)* of X as follows,

$$f(x) = \frac{g(x)}{\int_0^\infty g(u)du} \quad 0 \le x < \infty \tag{1}$$

A fundamental assumption of the detection function $g(x)$ is that it is monotonically decreasing with distance x from the transect line, meaning $g(x)$ decreases as x increases while satisfying $g(0)=1$. If $g(0)=1$then $f(0)=1/\int_0^\infty g(u)du$. Additionally, $g(x)$ is assumed to meet the shape criterion *(i.e.* $g'(0)=0$*)*, ensuring a “shoulder” at the origin. Consequently, the corresponding probability density function is also monotonically decreasing and has a shoulder condition at the origin.

Established the fundamental relationship for estimating the population density D of objects in a given area, expressed as,

$$D = \frac{E(n)f(0)}{2L}$$

where $E(n)$ is the expected number of detected objects, L is the

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total transect length, and f(0) is the probability density function of the perpendicular distance X at x=0.

This paper provides a comprehensive synthesis of methodological developments, interdisciplinary applications, and emerging directions in line transect sampling [2].

Theoretical Foundations of Line Transect Sampling

Let $X_1, X_2, \ldots, X_n$ be a random sample of perpendicular distances obtained using line transect technique. Burnham and Anderson (1976) provided the general estimate for D that stated by Equation (1). The general estimator of D is,

$$\hat{D} = \frac{n\hat{f}(0)}{2L} \qquad (2)$$

where $\hat{f}(0)$ is an appropriate sample estimator of $f(0)$ based on the observed perpendicular distances. Therefore, a key aspect of line transect sampling is the proper modeling of the detection function and the accurate estimation of $f(0)$.

Classical Line Transect Sampling

Classical models often assume a parametric detection function model for $g(x)$ and a corresponding pdf $f(x)$. A number of parametric models have been proposed for $f(x)$ in the literature. With one scale parameter, the classical exponential and the half normal models are the most prominent models [3]. Suggested the exponential model with pdf $f_1(x;\theta)=(1/\theta)e^{-x/\theta}, x\geq 0$. The corresponding detection function is $g_1(x;\theta)=e^{-x/\theta}$, which does not satisfy the shoulder condition assumption. Based on this model, the estimator of $f_1(0;\theta)$ by using the maximum likelihood method is $f_1(0;\hat{\theta})=1/\hat{\theta}$, where $\hat{\theta}$ is the mean of perpendicular distances $(i.e. \hat{\theta}=\bar{X})$. Consequently, Equation (2) leads to the estimator $\hat{D}=n/2L\bar{X}$ of D.

The half normal model that suggested by (satisfies the shoulder condition assumption), with detection function, $g_2(x;\sigma^2) = e^{-x^2/2\sigma^2}$, $x\geq 0$ and corresponding pdf $f_2(x;\sigma^2) = (2/\sigma\sqrt{2\pi})\, e^{-x^2/2\sigma^2}$. The maximum likelihood estimator of $f_2(0;\sigma_2)$ is $f_2(0;\hat{\sigma}^2) = (2/\pi T)^{1/2}$, where $T=\sum_{(i=1)}^{n} X_i^2/n$ [5]. Therefore, the estimator of D is $\hat{D}=n/(\sqrt{2\pi T}\, L)$. Quinn and Gallucci (1980) showed that the minimum variance unbiased estimator of $f_2(0;\sigma^2)$ is $\hat{f}_{2,MV}(0)=\frac{1}{\beta(n)}\left(\frac{2}{\pi\hat{\sigma}^2}\right)^{1/2}$

where $\beta(n)=\frac{\Gamma((n-1)/2)}{\Gamma(n/2)}\left(\frac{n}{2}\right)^{1/2}$ Zhang (2009) proposed the shrinkage estimator of $f_2(0;\sigma^2)$ under the half normal model, which is given by $f_{SH}(0)=\frac{n-2}{n}\beta(n)\left(\frac{2}{\pi\hat{\sigma}^2}\right)^{1/2}$ such as half-normal, exponential, Weibull, or Burr XII, but these assumptions may fail under heterogeneous conditions or violation of the shoulder condition. Nonparametric, kernel-based, histogram, and semi-parametric methods have been developed to address these limitations. Eidous's contributions include adaptive bandwidth selection, higher-order kernels, double-kernel estimators, bias-corrected histograms, and two-parameter detection functions, enhancing flexibility and robustness [6-15].

Many parametric models have been suggested to describe $f(x)$, and maximum likelihood methods have been widely used to estimate $f(0)$. These methods have been studied by many researchers, including [16-37].

Because accurate estimation of population density is important, researchers have suggested improving the form of the detection function. Field data show that different applications require different models to achieve a good fit.

Let $g(x)$ denote the detection function, representing the probability that an object at perpendicular distance x is detected. Classical parametric choices for $g(x)$ include half-normal, exponential, Weibull, or Burr XII. Weighted and mixture models further accommodate heterogeneous detection probabilities [38-46].

Nonparametric Kernel and Histogram Estimators

Nonparametric and kernel-based approaches overcome limitations of strict parametric assumptions. Kernel density estimators smooth observed distances $x_1, x_2, \ldots, x_n$ as:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right)$$

where K is a kernel function and h a smoothing bandwidth. Introduced variable-location and higher-order kernels, adaptive bandwidths, and bias corrections. Histogram-based estimators, including bias-corrected and additive histogram models, provide simple yet robust alternatives [16-20].

Semi-Parametric Models

Semi-parametric approaches combine parametric detection functions with nonparametric smoothing, achieving a balance between efficiency and flexibility. They are particularly effective when detection patterns deviate from classical assumptions or when censored or grouped data arise.

Methodological Advances

Weighted and Bias-Corrected Estimators

Weighted exponential models and bias-corrected histograms improve estimation under heterogeneous detection probabilities and imperfect shoulder conditions [20-25].

Fourth-Order Kernel and Double-Kernel Methods

Fourth-order kernels reduce bias and improve convergence [19-22]. Double-kernel methods employ two smoothing parameters to handle heterogeneous detection functions, particularly in censored or grouped data.

Two-Parameter Detection Functions

Two-parameter detection functions enhance flexibility, especially under non-detection at larger distances or shoulder-effect violations, improving density estimation in ecological surveys [17-20].

Interdisciplinary Applications

Ecology and Wildlife Management

Line transect and kernel-based methods remain essential for estimating animal abundances across mammals, birds, and marine species, allowing accurate density estimates under heterogeneous detection.

Environmental and Marine Sciences

Transect-based estimators evaluate pollutant concentration, vegetation, and seabed features, using kernel smoothing to capture fine-scale spatial variation.

Epidemiology and Public Health

Detection-based frameworks estimate spatial disease intensity, identifying clusters and transmission corridors via kernel-smoothed maps of infection density.

Industrial and Materials Sciences

Defect detection along continuous materials, fibers, or pipelines is modeled using line transect analogs, with kernel estimators improving quality control and spatial reliability assessment.

Recent Developments and Future Directions

Key developments include adaptive bandwidth selection, combined parametric-nonparametric estimators, bias correction, and high-dimensional extensions. Future research may integrate line transect principles with:

- Machine learning for automated detection function selection;
- Deep learning for spatially adaptive kernel smoothing;
- Three-dimensional and volumetric applications;
- High-resolution environmental and industrial data.

Conclusion

Line transect sampling has evolved from a tool for wildlife density estimation into a versatile statistical framework applicable across multiple scientific domains. Kernel-based, histogram, and semi-parametric innovations by Eidous and collaborators have enhanced robustness, flexibility, and applicability. These methods provide unified approaches for estimating densities and spatial distributions under imperfect detection, forming a foundation for future interdisciplinary research and computational integration.

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